

**An original advanced mathematical model of hemodynamics developed by
G.Poyedintsev - O.Voronova**

An original research into the field of hydrodynamics, which led to the development of the theory of the "third" mode of fluid flow, was carried out in the 1970s by Russian Scientist G.M. Poyedintsev (1929÷2006). Since 1978 Olga Voronova joined the Poyeditsev's Research Team to study hydrodynamics and human hemodynamics. Using a fresh axiomatic method, an analysis of prerequisites forming the groundwork for formulation of the classical equations of viscous fluid motion (the Navier-Stokes equations) was completed. That allowed improving the system of the axioms and, on its basis, creating an advanced mathematical model of the real fluid motion in a pipe, which adequately described the laminar and turbulent flows. Besides, the obtained solutions show that at the onset of a motion, the real fluid possesses a property of elevated fluidity and for a few fractions of a second demonstrates a specific flow pattern. In case of a two-phase fluid, similar to blood, in the initial phase of the accelerating of the fluid flow from rest, the flow structuring takes place.

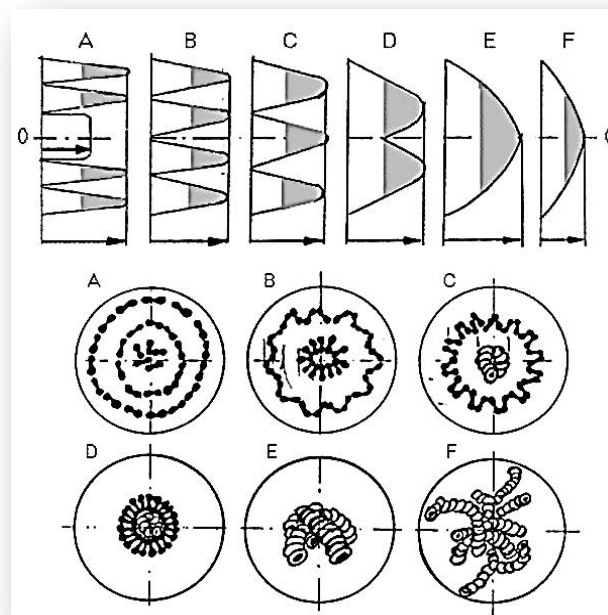


Fig.1. Formation of the specific flow pattern of a two-phase fluid in a pipe at the onset of a motion from rest (according to the mathematical model developed by

Figure 1 shows the location of erythrocytes in the blood stream, which corresponds to each instant of the time of the accelerating flow of fluid in a pipe from the state of rest before its transformation into the Poiseuille flow. While at the initial stage of the accelerating flow (in the "third" mode of flow), the concentric ring-type layers formed by alternating rings of blood elements and plasma are observed, in the laminar flow mode (items E and F in Figure 1 herein above) all the blood elements are found to be concentrated in the center of the stream. At the same time, the blood elements are contacting each other, forming a dense mass. It may result in an aggregation of erythrocytes and hemolysis. To avoid such pathological effects, it is necessary to control the proper blood flow structure, as described above, avoiding the loss of the number of the structured concentric layers in blood with time and maintaining the number of the concentric rings at the same level.

Following this way of the conceptual interpretation, the method of maintaining the fluid flow mode in a pipe with a stationary wave axially symmetric velocity profile and a static pressure for unbounded time was identified [1, 2].

It can be realized in a pulsating regime only. The most efficient mode of flow is the pulsating fluid motion along the elastic pipe wall, and in this case the fluid velocity and the pipe radius must change in each pulse according to certain strictly defined laws.

In terms of mathematics, it can be formulated as follows:

- the instantaneous velocity of the fluid flow motion along the elastic pipe changes in a pulse according to the following law:

$$U_t = U_0 \left(\frac{t - t_0}{t_1 - t_0} \right)^{0.4} \quad (1)$$

- the instantaneous radius of the elastic pipe lumen under the pipe expanding conditions varies according to the following law:

$$r_{+t} = r_0 \left(\frac{t - t_0}{t_1 - t_0} \right)^{0.2} \quad \text{при } t_0 \leq t \leq t_1 \quad (2)$$

- the instantaneous radius of the elastic pipe lumen under the pipe contraction

conditions varies according to the given law:

$$r_t = r_0 \left\{ \varepsilon + \frac{\beta}{2} \left[\varepsilon - \left(\frac{t}{t_0} \right)^{0,2} \right] \right\} \quad \text{при } t_1 \leq t \leq t_2 \quad (3)$$

- the maximum velocity of the fluid flow along the elastic pipe in a pulse is as follows:

$$U_0 = 37,5gt_0 \left[\frac{(5\varepsilon - 2)^3 - 27}{(5\varepsilon - 2)^5 - 243} \right] \quad (4)$$

Here:

$$\varepsilon = \left(\frac{t_1}{t_0} \right)^{0,2} = \left(1 + \frac{\Delta t_1}{t_0} \right)^{0,2} \quad (5)$$

$$\alpha = \left(\frac{t_2}{t_0} \right)^{0,2} = \left(1 + \frac{\Delta t_1 + \Delta t_2}{t_0} \right)^{0,2} \quad (6)$$

$$\beta = \frac{2(\varepsilon - 1)}{\alpha - \varepsilon} \quad (7)$$

t - current time; t_0 - the time of a pressure rise in the supply system to reach the level of the pressure available in the elastic pipe; Δt_1 - time of expansion of the elastic pipe in a pulse; Δt_2 - time of contraction of the elastic pipe in a pulse; g is gravity acceleration ($g = 9.81 \text{ m / s}$).

Based on the laws described by the above equations (1) ÷ (7), we can derive further formulas for calculating volume parameters of the fluid flow, namely:

Q1 is the volume of fluid entering the elastic pipe, delivered from the supply system during the time of its expansion;

Q2 is the volume of fluid entering the elastic pipe, delivered from the supply system during the pipe contraction;

Q3 is the volume of fluid filling an increase in the elastic pipe internal volume during the pipe expansion. This specific volume is ejected during the pipe contraction due to the pipe elastic properties. In other words, the elastic pipe operates as a peristaltic pump.

Following that way, an advanced mathematical model of an optimal hydraulic system for the transportation of multi-phase fluids, similar to blood, organized in the most efficient "third" flow mode, was developed.

At further stages of the model development and validation, a complete set of attributes, characterizing this hydraulic system in most complete and accurate manner, was identified in the circulatory system of a human. This has produced evidence that the blood circulation in a human organism is provided not under the Poiseuille flow conditions, but in the most efficient "third" flow regime, characterized by the specific, axially symmetric, concentrically "layered" pattern, when the blood cells move with a velocity exceeding the average flow one [1, 2].

Results

The above mentioned constructed mathematical model of the optimal hydraulic system performance was adapted to describe the processes taking place in the circulatory system in a human organism. Let us present an algorithm for solving the given problem in more details.

- As a theoretical basis, the mathematical model of the optimal hydraulic system has been applied [1]. The model contains the mathematical laws of fluid flow in the third mode along the elastic pipe, which are described by the equations (1)÷(7). The formulas for determining the systolic and diastolic volume parameters of the central hemodynamics in a human with the use of the durations of the respective cardiac cycle phases have been derived.
- Electrocardiography has been used as a method to determine the phase structure of a cardiac cycle [3].

The initial data to calculate the volumetric hemodynamic parameters are the respective durations of the ECG waves and segments, namely:

QRS - the duration of the complex taken from the Q wave onset to the S wave end;

RS - the duration of the complex taken from the R wave peak to the S wave end;

QT - the duration of the Q-T interval (from the Q wave onset to the T wave end);

PQ is the duration of the P-Q segment (from the P wave end to the Q wave onset);

TT is the duration of a cardiac cycle, measured from the end of the T wave to the end of the next T wave.

The performance of the heart and blood vessels in systole and diastole is characterized by the following hemodynamic parameters:

SV - stroke volume of blood (ml);

MV - minute volume of blood circulation (l / min);

PV1 and PV2 are the volumes of blood entering the heart ventricle in the phase of the early diastole and the atrial systole, respectively (ml);

PV3 and PV4 are the volumes of blood ejected by the heart ventricle in the phases of the rapid and slow ejection, respectively (ml);

PV5 is the volume of blood pumped by the ascending aorta, operating as a peristaltic pump, during the systole phase (ml).

Let us briefly describe an algorithm for determining the systolic volumetric parameters of hemodynamics using the durations of the respective cardiac cycle phases.

- The sectional area of the blood vessel at the ascending aorta level is taken into account.
- S_0 is the aortic sectional area (cm^2), determined according to the applicable nomogram or by any other equivalent method.
- Duration t_0 is defined as follows:

$$t_0 = RS \quad (\text{s})$$

- The systole duration (including t_0) is described by the equation as follows:

$$t_2 = QT - QRS + RS \quad (\text{s})$$

- The heart rate is determined as given below:

$$HR = 60 / TT \quad (\text{beats per min})$$

- Dimensionless parameters and functions thereof are as indicated below:

$$\alpha = \left(\frac{t_2}{t_0} \right)^{0,2} \quad \varepsilon = \frac{3\alpha + 2}{5} \quad \beta = \frac{2(\varepsilon - 1)}{\alpha - \varepsilon}$$

$$f_1(\varepsilon, \alpha, \beta) = \frac{5}{12} \varepsilon^2 (2 + \beta)^2 (\alpha^3 - \varepsilon^3) - \frac{5}{8} \varepsilon \beta (2 + \beta) (\alpha^4 - \varepsilon^4) + \frac{1}{4} \beta^2 (\alpha^5 - \varepsilon^5)$$

$$f_2(\varepsilon) = \varepsilon^5 - \frac{5}{3} \varepsilon^3 + \frac{2}{3}$$

- The maximum velocity of the blood flow in a pulse is as follows:

$$U_0 = \frac{36787,5 t_0 [(5\varepsilon - 2)^3 - 27]}{(5\varepsilon - 2)^5 - 243} \quad (\text{cm/s})$$

- The volume of blood ejected by the heart ventricle during the rapid ejection phase is determined as given below:

$$PV3 = S_0 * U_0 * t_0 (\varepsilon^5 - 1) \quad (\text{ml})$$

- The volume of blood ejected by the heart ventricle during the slow ejection phase is calculated as follows:

$$PV4 = S_0 * U_0 * t_0 [f_1(\varepsilon, \alpha, \beta) - f_2(\varepsilon)] \quad (\text{ml})$$

- The volume of blood pumped by the ascending aorta in the systole, characterizing the aortic tonus, is computed as given below:

$$PV5 = S_0 * U_0 * t_0 * f_2(\varepsilon) \quad (\text{ml})$$

- The stroke volume of blood is calculated according to the following equation:

$$SV = PV3 + PV4 \quad (\text{ml})$$

- The minute volume of blood circulation is computed as given below:

$$MV = SV * HR / 10^3 \quad (1)$$

- The specific stroke blood volume referred to the unit of the ascending aorta lumen area (the given parameter is required to calculate the diastolic phase volumes) is calculated as follows:

$$SSV = SV / S_0, \quad (\text{ml/cm}^2)$$

Now let us give a brief description of an algorithm for determining the diastolic volume parameters of hemodynamics.

- The diastole is considered as two successive systoles, during which blood is

ejected from the atrium into the ventricle (the early diastole and the atrial systole).

- Durations t_{01} and t_{02} are determined theoretically, provided that the blood structure is maintained at the time of the transition from the phase of the early diastole to the atrial systole. It has been evidenced that this is valid under the following condition:

$$t_{01} = t_{02}$$

- The duration of the early diastole period (including t_{01}) is as follows:

$$t_{21} = TT - QT - PQ$$

- The duration of the atrial systole period (including t_{02}) is calculated as given below:

$$t_{12} = t_{02} + PQ = t_{01} + PQ$$

- Dimensionless parameters and functions thereof are computed as follows:

$$\alpha_1 = \left(\frac{t_{21}}{t_{01}} \right)^{0,2} \quad \varepsilon_1 = \frac{3\alpha_1 + 2}{5} \quad \beta_1 = \frac{2(\varepsilon_1 - 1)}{\alpha_1 - \varepsilon_1} \quad \varepsilon_2 = \left(\frac{t_{12}}{t_{01}} \right)^{0,2}$$

$$f_1(\varepsilon_1, \alpha_1, \beta_1) = \frac{5}{12} \varepsilon_1^2 (2 + \beta_1)^2 (\alpha_1^3 - \varepsilon_1^3) - \frac{5}{8} \varepsilon_1 \beta_1 (2 + \beta_1) (\alpha_1^4 - \varepsilon_1^4) + \frac{1}{4} \beta_1^2 (\alpha_1^5 - \varepsilon_1^5)$$

$$f_2(\varepsilon_1) = \varepsilon_1^5 - \frac{5}{3} \varepsilon_1^3 + \frac{2}{3}$$

- Maximum velocity U_{01} for the early diastole period and U_{02} for the atrial systole is the following:

$$U_{01} = \frac{36787,5 t_{01} \left[(5\varepsilon_1 - 2)^3 - 27 \right]}{\left(5\varepsilon_1 - 2 \right)^5 - 243} \quad (\text{cm/s})$$

Velocity U_{02} is calculated using the same formula at $\varepsilon = \varepsilon_2$.

- The time interval t_{01} is determined based on a certain specified condition. To calculate it, we take value t_{01} , which satisfies the following equation:

$$\begin{aligned}
SSV - t_{01} \{ & U_{01} [\varepsilon_1^5 - 1 + f_1(\varepsilon_1, \alpha_1, \beta_1) - f_2(\varepsilon_1)] \\
& + f_1(\varepsilon_1, \alpha_1, \beta_1) - f_2(\varepsilon_1) \} + U_{02} (\varepsilon_2^5 - 1) \} \\
& = 0
\end{aligned}$$

- The RV1 blood volume, entering the heart ventricle in the phase of the early diastole, expressed as a percentage of the filling volume, is computed as indicated below:

$$RV1 = \frac{100 * U_{01} [\varepsilon_1^5 - 1 + f_1(\varepsilon_1, \alpha_1, \beta_1) - f_2(\varepsilon_1)]}{U_{01} [\varepsilon_1^5 - 1 + f_1(\varepsilon_1, \alpha_1, \beta_1) - f_2(\varepsilon_1)] + U_{02} (\varepsilon_2^5 - 1)} \quad (\%)$$

- The RV2 blood volume, entering the heart ventricle in the phase of the atrial systole, expressed as a percentage of the filling volume, is determined as follows :

$$RV2 = 100 - RV1 \quad (\%)$$

- Basing on the condition to maintain the balance between the blood input during the diastole and the output during the systole, the condition of the equality of the volumes required for filling and ejection can be accepted. Then, knowing the stroke volume SV value, it is possible to determine the absolute (ml) values of diastolic phase volumes PV1 and PV2:

$$PV1 = SV * RV1 / 100 \quad (\text{ml})$$

$$PV2 = SV * RV2 / 100 \quad (\text{ml})$$

In the studies, the relative phase volumes of blood (RV1 ÷ RV5), representing a percentage of the absolute value of the respective phase volume referred to the stroke volume SV, were also calculated and analyzed. The hemodynamic parameters, the values of which are expressed as a percentage of the stroke volume, characterize the individual contribution of each phase process to the formation of cardiac output.

Conclusions

- As a theoretical basis for the research into the peculiarities of the circulatory system performance, an original mathematical model of the optimal hydraulic system for transportation of multi-phase fluids, similar to human blood, in the

most efficient "third" flow regime, has been applied.

- The advanced mathematical model of hemodynamics developed by G. Poedintsev - O. Voronova, describing the main laws of the transportation function in the circulatory system, has been created.
- A new non-invasive methodology and technology for measuring volumetric phase-related parameters of the human central hemodynamics, with the use of the cardiac cycle phase durations, identified on ECG, have been developed. These theoretical ideas to measure non-invasively hemodynamics in a human have been successfully translated into practice: a new PC-assisted medical device, patented under the name Cardiocode, has been designed and manufactured.
- The volumetric phase-related hemodynamic parameters are actually the most informative markers of the human circulatory system performance, since they reflect an integrated operation of the heart and blood vessels. Knowing their individual contributions, their interplay and the relations between the anatomical configuration and the functional profile of the heart and blood vessel performance, it is possible to make most perfect and reliable diagnostics of the circulatory system in a human and reveal pathology, if any. The developed technology and equipment can be successfully used to assess efficacy both of medication and prevention.